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DEPARTMENT OF THE ARMY
Fort Detrick
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#249.
9.2

2. THE EQUATIONS OF MOVEMENTS FOR DUST PARTICLES

a. The laws of resistance.

For the movement of a particle of dust under the influence of any arbitrary power or for the resistance of a particle held fast during a given speed, the proper equations may be applied. For the resistance W (kg.) of a body, facing the current with its surface F (m^2) we have the equation:

$$W = cqF. \tag{1}$$

The pressure of the dust is in this equation q (kg/m^2) and represents a function of the speed of the current v (m/s) and of the specific gravity γ gamma (kg/m^3) of the air carrier with an acceleration of gravity.

$$g = 9.81 \ m/sec^2$$
$$q = \frac{v^2 \gamma}{2g} \tag{2}$$

The number of resistance c which has no dimension and depends upon the speed, the viscosity of the air (Gaszähigkeit?) and the dimension of the dust particle C has always the same value for similar currents. A similarity of currents is present if the Reynold's figure R has the same value. For this reason, c is properly applied, depending upon R according to the equation $R = \frac{vd}{\nu}$. In this equation d (m) signifies any characteristic measurment of the body of resistance; for example, diameter or length and ν (m^2/s) the kinetic viscosity of air. (3)

The figure c of resistance is sufficiently known for a large number of forms of bodies and for a wide area of R, thus for example, for bullets by the tests made by Allen, Schmiedel, Liebster and by the values

found in Göttingen. There is also a number of values available for circular flat discs, namely those of Schmidel and of the Experimental Institute of Göttingen. The values known for other forms of bodies, however, lie in a realm of R, which is out of the question for the movement of dust.

Contrary to other applications, whereby R is very large, values of R are considered for the movement of dust which are smaller than 10 and frequently considerably smaller on account of the size of the dust particles. A relatively equal relation may be assumed for this realm without a great error, thus, the value for bullets corresponds to $c = \frac{24}{R}$. (4)

This formula is known in the technique of dust in a different meaning than the Stokes' law, by which the velocity of gravitation is calculated.

Later an extension has been indicated by Oseen; namely

$$c = \frac{24}{R} + 4.5 \qquad (5)$$

Stokes' law, however, is the base of the following calculations as it is sufficiently exact, leading to relatively simple equations of movements.

b. Derivation of the equations of movements and application of analogical mechanics.

Vertical invasion into an equidistant area of currents. Let us look at first an area of current with equidistant and straight current lines and with the velocity of w_0 according to fig. 2. At the starting point

of the coordinates a dust granule of spherical shape and a diameter of d is shot into this area at the velocity of v_0 and at the time of $t = 0$. By the velocity w_0 a deflection and acceleration of the dust particle takes place in the direction of the positive y-axis; while the resistance of the air retards the movement in the direction of x. Under these two influences, there is a continuously arched course for the dust particle. The movements in the direction of the two axes may be considered first individually and then overlapping one another. For the direction of x the formula $-m dv = W\,dt$ may be applied. (6)
In this formula, m stands for the mass of the dust particle and W for the air resistance.

From equations (1), (2), (3) and (4) we obtain $W = av$ (7)

whereby $a = 6\,r\,\pi\,\eta$ (8)

In this formula r (m) is the radius of the ball and η (kg s/m²) the viscosity of the air or dust carrier. After one integration, the components of velocity in the direction of x are obtained

$$v_x = v_0 e^{-\frac{a}{m} \cdot t} \tag{9}$$

Accordingly, a decrease in the velocity from the original value v_0 towards zero takes place.

In order to ascertain the course in the direction of x as a function of the time under the prevailing conditions of limitation another integration is required:

$$x = \frac{m}{a}\, v_0 \left(1 - e^{-\frac{a}{m} t}\right) \tag{10}$$

or

$$x = \frac{m}{a}\,(v_0 - v_x) \tag{10a}$$

The value for the limit of x may be calculated from this and which for $t = \infty$ or $v_x = 0$

$$x_{\infty} = \frac{m}{a} v_o \tag{11}$$

At the initial speed v_o of the vertical invasion, the dust particle proceeds in the direction of x for the most about this distance; because the energy of invasion is exhausted by the air resistance. The value of x_{∞} enters the calculations later and may be designated as the extent of deceleration to the signification of which we shall return to report more in detail.

Similar considerations may be employed also for the direction of y. The dust particle entering at the zero point of the system of coordinates is exposed at once to the velocity of the air w_o by which it is accelerated up to the same value. Also in this case, the formula as in equation (6) may be employed, whereby, the relative velocity between particle and air is conclusive.

$$W = a\,(w_o - v_y) \tag{12}$$

The differential equation for the movement reads therefore:

$$\frac{dv}{w_o - v} = -\frac{a}{m}\,dt \tag{13}$$

The integration shows the velocity v_y in the direction of the ordinates.

$$v_y = w_o \left(1 - e^{-\frac{a}{m} t}\right) \tag{14}$$

according to the parenthesis

This equation means that ~~after the invasion~~ of the particle an acceleration takes place in the direction of y , which acceleration lasts until $v_y = w_o$. Theoretically this will take place after a considerably long time; practically, however, in a relatively short time.

Another integration produces the values of y in relation to ~~depending upon the~~ time t:

$$y = w_0 \left[t - \frac{m}{a} \left(e^{-\frac{a}{m}t} - 1\right)\right] \tag{15}$$

With the aforesaid equations, the position and the velocity of the dust particle may be ascertained for each time period. The absolute speed is given ~~obtained~~ by the geometric addition of the two components ~~...~~

$$v_r = \sqrt{v_x^2 + v_y^2} \tag{16}$$

After the introduction of the extent of deceleration (Bremsweg), the coordinates x and y are:

$$x = x_{00} \left(1 - e^{-\frac{a}{m}t}\right) \tag{17}$$

$$y = y_{00} \left[\frac{a}{m}t - \left(e^{-\frac{a}{m}t} - 1\right)\right] \tag{18}$$

In technical problems of the movement of dust in streaming ~~areas of~~ currents, one ~~it~~ rarely deals with such a simple relationship ~~happens~~ as indicated in fig. 2. The formulae therefore call for a generalization.

<u>Generalization of the fundamental equations.</u> The area of the current with the speed of w_0 is assumed to be again equidistant and straight fig. 3. The incline of the lines of the current to the axis ξ corresponds to the angle δ; whereas, the dust particle invades at an angle β of incline with velocity v_0 at the starting point of the coordinates. After according to a few preliminary ~~intermittent~~ calculations, the equations of ~~the~~ movement for ~~the~~ velocity and ~~the~~ direction in relation to ~~depending upon the~~ time t are as follows:

$$v_\xi = w_0 \cos\alpha\left(1 - e^{-\frac{a}{m}t}\right) + v_0 \cos\beta\, e^{-\frac{a}{m}t} \tag{19}$$

$$v_\eta = w_0 \sin\alpha\left(1 - e^{-\frac{a}{m}t}\right) + v_0 \sin\beta\, e^{-\frac{a}{m}t} \tag{20}$$

$$\xi = \frac{m}{a} w_0 \cos\alpha \left[\frac{a}{m}t - \left(1 - e^{-\frac{a}{m}t}\right)\right] + \frac{m}{a} v_0 \cos\beta\left(1 - e^{-\frac{a}{m}t}\right) \tag{21}$$

$$\eta = \frac{m}{a} w_0 \sin\alpha \left[\frac{a}{m}t - \left(1 - e^{-\frac{a}{m}t}\right)\right] + \frac{m}{a} v_0 \sin\beta\left(1 - e^{-\frac{a}{m}t}\right) \tag{22}$$

The absolute final velocity is in this case

$$v_r = \sqrt{v^2 + v^2} \tag{23}$$

furthermore, the incline of the course of the dust particle towards the ξ axis is $tg\beta' = \frac{v_\eta}{v_\xi}$ (24)

The aforesaid equations permit a point by point uniform calculation of the courses of dust particles in given moving air ~~areas of~~ currents under the presumption of sufficiently small enclosed ~~intermittent~~ spaces for which the current ~~is~~ to be considered are unalterable and parallel.

Certain presumptions have to be made in these calculations with regard to the dust particle, its diameter, its specific gravity; and also to the velocity of the air current and its viscosity, tenacity. These factors of influence are contained, however, partially in the clearly marked ~~extent~~ of deceleration, so that the latter enters as a characteristic value ~~is fitting~~ into further calculations.

<u>The extent of deceleration</u> (Bremsweg). The basic concept ~~conceptive determination~~ is given ~~already~~ by the equation 11. If the mass of the ball and the index of the logarithm a of Stokes' law are placed according to equation 8,

it follows that with the specific weight of the dust γst (kg/m^3)

$$\frac{m}{a} = \frac{r^2 \gamma st}{4.5 g \eta} \qquad (25)$$

The factor $\frac{m}{a}$ is according to equation 11, the specific extent of deceleration x'_{oo}, with reference to the velocity v = 1 m/s.

Assuming an air temperature of 20° C. and a specific weight of 1000 kg./m^3 for the dust ball, the specific extent of deceleration x'_{oo} is represented in fig. 4 depending upon the diameter of the ball.

The velocity of falling under the influence of the force of gravity in a space free from all current may be put as a characteristic value for the dust particle on its way to the extent of deceleration or to the specific extent of deceleration in proportion. According to Stokes' law, the velocity of descend (falling) v_s:

$$v_s = \frac{mg}{a} \qquad (26)$$

The extent of deceleration consequently is $x_{oo} = \frac{v_s v}{g}$ (27)

In connection with this, it should be noted that the velocity of falling should be referred of course to those values, of which it is the question in a raised problem; in other words, to a definite viscosity of the air, etc. If experimental values with regard to the velocity of falling and values of calculation do not agree, a re-calculation according to equation 25 has to be made.

For a demonstration of the formulae deduced for the vertical invasion the existing velocities and coordinates of the locality of a spherical granule of dust of 50 μ diameter are calculated for the values given depending upon the time and are indicated in fig. 5. It is seen

that v_x decreases very rapidly towards the 0 value, whereas, the v_y r caches the value w_0. The existing values of x and y are included in is diagram; the progression in the direction of x centrifuges to the value of limitation x_{00} := 77 mm.

In order to show the influense of the diameter of the dust ball upon the course of the dust, the respective courses of the dust are indicated in fig. 6, under the same conditions, but for the diameters of 40,50 and 60 μ of the dust balls. The starting events take place more rapidly in particles of smaller diameters. The diagram also shows distinctly the values of the extent of deceleration and the approach of the dust courses to the latter.

Conceptive determination of the relative degree of dust elimination. The diagram in fig. 7 shows the air currents on the windward side for a cylinder of a d - diameter, furthermore, the dust currents for a given dust particle of ball shape and an assumed spec. gravity, etc. It is seen that only a part of the arriving dust particles are hitting the cylinder, whereas, the other take their course on the sides of the cylinder. The course of the particles, which hit the cylinder just tangentially are designated as border courses. The distance of these border courses b from one another at a sufficient distance from the cylinder, where they are still parallel to one another, and divided by the diameter of the cylinder, shows the relative degree of dust elimination e:

$$e = \frac{b}{d} \qquad (28)$$

The dependence of e upon the various influencing factors will be investigated somewhat more in detail in the subsequent chapter.

Consideration of analogy. The extent of deceleration contains already all independent changes, except the extension of the profile to be investigated. It is obvious, therefore, to determine the index of a logarithm without dimension in such a way that the characteristic size of an arbitrary profile is placed in proportion to it, as for example, the diameter of an unusually long cylinder, the width of a plate, the size of the opening in a catching apparatus, etc.

Accordingly, we have for   the formula: $= \frac{x_{oo}}{d}$ (29)

By a simple deliberation it may be ascertained that for a given profile the degree of dust elimination depends only upon this index. For example, for two unusually long cylinders of the diameter d_1 and d_2 which are present in areas of currents of the velocity of w_{o1} and w_{o2} the deflection of the dust particles be determined with the masses m_1 and m_2 perpendicular to the direction of the current. For the first cylinder we have:

$$\sum \frac{\Delta \xi_1}{x'_{oo_1}} = \sum w_1 \cos \delta_1 \left[\frac{a}{m} t_1 - \left(e^{-\frac{a}{m_1} t_1} - 1 \right) \right] + v_1 \cos \beta_1 \left(e^{-\frac{a}{m_1} t_1} - 1 \right) \quad (30)$$

Accordingly we have for the second cylinder the same formula but with the indices two. A similarity may be expected, if the following conditions are fulfilled:

$$\frac{a}{m_1} t_1 = \frac{a}{m_2} t_2 \; ; \; \delta_1 = \delta_2 \; ; \beta_1 = \beta_2 \quad (31)$$

Besides the lines of the current have to have a similar course for both cylinders, which is correct on the windward side for the potential current.

After further calculations we find that the only condition for the similarity of the courses of the dust particles the relations for $\frac{x_{oo}}{\varepsilon}$ have to be equal, in other words, the index of the logarithm determined according to the equation (29) must be the same.

If the independence of the currents from the velocity is no more indicated, as for example, when the currents on the lee side have to be taken into consideration then there is a limiting factor present, which requires that the Reynold's figure, as referred to the profile, equals the formula:

$$w_{o1}d_1 = w_{o2}d_2 \qquad (32)$$

The index of the logarithm ψ is connected also with the velocity of falling, exactly as the extent of deceleration. In its most general form it may be written as follows: $\psi = \frac{wv_s}{dg}$ (33)

After the foregoing reflections it suffices to find out the dependence of the relative degree of dust eliminati n as per formula (28) from the index of the logarithm according to formula (33) for the deposition of dust on the windward side.

C. Investigation on the deposition of dust on three forms of bodies.

1. Selection of forms.

The curvature in opposition to the direction of the current has been the principal point of view in our present investigation. The following three forms of bodies have been investigated individually, namely: a circular cylinder of 100 mm. diameter, a transversally placed plate of even thickness (fig. 8) and a catch-apparatus (fig. 9). The pictures of the currents, as per fig. 10 to 12 have been taken of these forms with the apparatus (fig. 1).

3. Performance of calculations.

The pictures of the currents are used first for the determination of lines of the same velocity and of the same incline of the currents. (Figs. 13 to 15). In order to obtain a satisfactory accuracy it is necessary to take as small as possible the existing intermedian spaces for the individual calculation of the courses of the dust. It has been found to be more suitable to equalize the time intervals, because the expressions in parenthesis in the equations of motion have then the same value. Arbitrary courses are calculated the first for each form of body, in order to obtain from them the inclusion of the courses of limitation are represented in this way in the form of points for different logarithms.

4. Extension of the results to currents of three dimensions.

The course of the area of the current for the potential current is sufficiently well known in connection with the ball formation, thus, the relative degree of the elimination of dust may be determined in the same way in dependence on the index of the logarithm as shown in fig. 19. The current and the calculation of thecourses of the dust has been carried out in a plain which coincided with the principal direction of the current. In this connection the relative degree of the elimination of dust results in $e = \frac{b^2}{d^2}$ (34)

The degrees of the elimination of dust are, of course, lower than in connection with the cylinder. If it is assumed that in the area of the current of a disk in relation to that of an equally thick plate (fig. 20) the same similarity exists as between the areas of the current of a ball -

and of a cylinder (fig. 19) a closer indication as to the relative degree of dust-elimination is shown in connection with a disc and a catching cylinder, (fig. 21). In this connection the catching cylinder is to be imagined in such a way that the transversal cut according to fig. 9 has formed the body, revolving around an axis in the direction of the current.